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# METHOD OF DETERMINING TEMPERATURE AND CALCULATION OF THERMAL STRESSES IN POROUS PERMEABLE WALLS

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UDC 543.54

The article suggests a method of determining temperature and of calculating thermal stresses in the range of elastic deformations and of the boundaries of the plastic zones in elastoplastic deformation of the material of a permeable cylindrical wall with porous cooling in steady-state heating.

When units of heat machines made of permeable materials with porous cooling are designed, the permissible temperature of the working surface is specified, and from it the required flow intensity of the coolant is determined. To save coolant and to reduce the weight of the machine, it is advantageous to have the permissible temperature of the working surface as high as possible, but then the reduced strength of the wall has to be taken into account, and this is the decisive factor in the evaluation of its durability. Consequently, the permissible temperature and the necessary flow rate of the coolant have to be chosen such that the thermal stresses in the wall do not exceed the permissible values.

The known methods of measuring thermal stresses (the optical method, the method of lacquer coatings, x-raying, strain-gauge methods) [1] are practically unsuitable in this case because the results of measurements are difficult to process and insufficiently accurate.

Good results in determining thermal stresses in a permeable wall are attained by a method based on measuring the temperature of both its surfaces [2]; however, when contact sensors are used for the purpose, a considerable error arises because of the local infringement of permeability of the material at the place of contact and becasue of large temperature gradients across the wall. Measuring the temperature of porous walls with contactless means also encounters great difficulties because they do not act rapidly enough and because the measuring apparatus is too complex.

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 0022-0841/85/4905-1334$09.50 

We suggest a new method of determining the temperature and the calculation of thermal stresses in permeable walls with porous cooling, based on the change of pressure of the coolant ahead of the porous wall in dependence on its thermal state. It was established experimentally and theoretically that when the structure of the material of the wall is fine-meshed, the heat transfer coefficient of the cooling gas with the wall is infinitely large. In consequence of this, the gas assumes the temperature of the porous structure practically already upon entering the wall [3], and since it becomes heated it expands volumetrically in proportion to the temperature state of the wall. This causes an increase in pressure of the coolant before it enters the porous wall.

One of the shapes of products made of permeable material with porous cooling of the wall is the cylinder. Of greatest interest is the study of its state of thermal stress in steady-state thermal regime of its heating. This regime is characterized by the largest temperature gradient across the wall, and this is decisive in the evaluation of the necessary flow rate of coolant.

From the solution of the equation describing the law of conservation of energy for unit length of a cylindrical wall in steady temperature state without taking radiation from the outer surface into account, a dependence was obtained which characterizes the temperature distribution across its thickness:

$$T = T_0 + (T_1 - T_0)\left(\frac{R_1}{r}\right)^{\nu}. \tag{1}$$

From formula (1) we obtain an expression for the temperature $T_1$ of the inner surface through the temperature $\bar{T}$, the average along the radius of the section of the porous wall which does not differ from the mean integral wall temperature by more than 2-3%, i.e.,

$$T_1 = T_0 + (\bar{T} - T_0)\left(\frac{R_1 + R_2}{2R_1}\right)^{\nu}. \tag{2}$$

When the porous permeable wall is in the isothermal state, its temperature, the pressure of the coolant, and the hydraulic friction factor are correlated with each other by the dependence [4]

$$\xi_T = \frac{P_1^2 - P_2^2}{\beta j^2 \bar{R} T h}. \tag{3}$$

When there is no dissociation of the coolant, the hydraulic friction factor is practically not dependent on the temperature regime of the flow at all. For instance, when gaseous helium flows through a porous nonisothermal tungsten wall, the correction coefficient for nonisothermy is ≅1.02; we may therefore take it that with $\bar{T} = T_{\text{isotherm}}$

$$\xi_{\bar{T}} \simeq \xi_T. \tag{4}$$

Moreover, the hydraulic friction factor of porous walls made of cermet materials changes only slightly, approximately by up to 10%, when the temperature increases to the temperature of dissociation of the coolant. In that case,

$$\xi_{T_0} \simeq \xi_{\bar{T}} \simeq \frac{P_1^2 - P_2^2}{\beta j^2 \bar{R} \bar{T} h}. \tag{5}$$

Dependence (1) characterizing the temperature distribution can be expressed with a view to (2)-(5) through the change of pressure of the coolant caused by the heating of the porous wall, i.e.,

$$T = T_0\left[1 + \frac{P_1^2 - P_0^2}{P_0^2 - P_2^2}\left(\frac{R_1 + R_2}{2r}\right)^{\nu}\right]. \tag{6}$$

Expression (6) describes the temperature field in steady-state heating regime; if we substitute it into the dependences correlating the thermal stresses with the temperature distribution across the wall [5], we obtain formulas for determining the thermal stresses arising in a hollow permeable cylinder upon thermal loading of the inner surface with porous cooling of the wall.

The dependences obtained for the state of plane deformation of a porous permeable wall have the form

$$\sigma^t_{r(r)} = \frac{\alpha EAT_0}{r^2(1-\mu)(2-\gamma)}\left(\frac{R_1+R_2}{2}\right)^{\gamma}\left[\frac{R_2^{2-\gamma}(r^2-R_1^2)-R_2^{2-\gamma}(r^2-R_2^2)}{R_2^2-R_1^2}-r^{2-\gamma}\right], \quad (7)$$

$$\sigma^t_{\theta(r)} = \frac{\alpha EAT_0}{r^2(1-\mu)(2-\gamma)}\left(\frac{R_1+R_2}{2}\right)^{\gamma}\left[\frac{R_2^{2-\gamma}(r^2+R_1^2)-R_1^{2-\gamma}(r^2+R_2^2)}{R_2^2-R_1^2}-r^{2-\gamma}(1-\gamma)\right], \quad (8)$$

$$\sigma^t_{z(r)} = \frac{\alpha ET_0}{1-\mu}\left[A\left(\frac{R_1+R_2}{2}\right)^{\gamma}\left(\frac{2\mu}{2-\gamma}\;\frac{R_2^{2-\gamma}-R_1^{2-\gamma}}{R_2^2-R_1^2}-r^{-\gamma}\right)-1+\mu\right], \quad (9)$$

and when there is no load on the end faces,

$$\sigma^t_{z(r)} = \frac{\alpha EAT_0}{(1-\mu)(2-\gamma)}\left(\frac{R_1+R_2}{2}\right)^{\gamma}\left[\frac{2(R_2^{2-\gamma}-R_1^{2-\gamma})}{R_2^2-R_1^2}-r^{-\gamma}(2-\gamma)\right]. \quad (10)$$

In calculating the thermal stresses in permeable walls by the presented dependences, tabulated values of thermal conductivity, linear expansion, modulus of elasticity, and Poisson ratio of porous material are used for a temperature that is determined from the expression

$$\overline{T} = \frac{P_1^2-P_2^2}{P_0^2-P_2^2}T_0. \quad (11)$$

For porous permeable walls where the hydraulic friction factor depends on the temperature, formulas (7)-(10) differ only by the parameter A. In this case we use the tabulated values of the coefficients α, E, μ, and λ at the mean wall temperature, determined, e.g., from the condition of linear change of the value of $(P_1^2 - P_2^2)/T$, characterizing its hydraulic friction factor, to calculate the stresses in dependence on the temperature, i.e.,

$$\frac{P_1^2-P_2^2}{\overline{T}} = K(\overline{T}-T_0)+\frac{P_0^2-P_2^2}{T_0}, \quad (12)$$

and hence

$$\overline{T} = \frac{KT_0^2-P_0^2+P_2^2\pm\sqrt{(KT_0^2-P_0^2+P_2^2)^2+4KT_0^2(P_1^2-P_2^2)}}{2KT_0}. \quad (13)$$

From an analysis of dependences (5), (11), (7)-(10) it can be seen that

$$A = \frac{KT_0^2-P_0^2+P_2^2\pm\sqrt{(KT_0^2-P_0^2+P_2^2)^2+4KT_0^2(P_1^2-P_2^2)}}{2KT_0^2}-1. \quad (14)$$

The coefficient K is determined from the temperature dependence describing the nature of the change in the hydraulic friction factor of the wall. Thus, if the dependence $\xi_T = f(T)$ is of linear nature, then the coefficient K is found from expression (12) according to the results of determining $(P^2 - P_2^2)/T$ at two arbitrary wall temperatures for the given flow rate of coolant.

In dependence on the magnitude of the external loads and the temperature gradient across the wall thickness, the material of the wall is in an elastic or elastoplastic state. With increasing mechanical loads or temperature factors the zones of plastic deformation across the wall increase. With a view to the results of the investigations of [6, 7] the boundaries of the plastic zones in steady thermal state of the wall of a permeable cylinder with porous cooling are determined from the expression

$$r_c,\ r_d = \left[\frac{R_2^{2-\gamma}-R_1^{2-\gamma}}{(1-\gamma)(R_2^2-R_1^2)}\pm\frac{\sigma_{T(T)}(1-\mu)(2R_1R_2)^{\gamma}}{\alpha EA(R_1+R_2)^{\gamma}}\right]^{-\frac{1}{\gamma}}. \quad (15)$$

The suggested dependences are correct for permeable cylinders with a porosity not exceeding 30% for which $\ell \geq 2.4\sqrt{R_1(R_2 - R_1)}$ and the mechanical characteristics change only slightly with a change of temperature (materials of the type of porous tungsten).

## NOTATION

$T_0$, wall temperature before thermal loading, °K; $T_1$, $T_2$, temperature of the inner and outer surfaces, respectively, of a porous wall, °K; r, radial coordinate, m; $R_1$, $R_2$, inner and outer radius, respectively, of a porous wall, m; m, flow rate of coolant, kg/(m·sec); j, flow rate of coolant, kg/($m^2$·sec); $c_p$, heat capacity of the coolant, J/(kg·deg); P, pressure, N/$m^2$; $P_0$, $P_1$, pressure of coolant in front of the porous wall in the cold and hot state, respectively, N/$m^2$; $P_2$, pressure of the coolant at the outlet from the porous wall, N/$m^2$; $\lambda_w$, thermal conductivity of the material of the wall, W/(m·deg); $\gamma$, $(mc_p)/(2\pi\lambda_w)$; A (for $\xi_{T_0} = \xi_{\bar{T}}$) $-(P_1^2 - P_0^2)/(P_0^2 - P_2^2)$; $\alpha$, coefficient of linear expansion, 1/deg; E, Young's modulus, kgf/$mm^2$; $\mu$, Poisson ratio; $\sigma_T(T)$, yield strength, kgf/$mm^2$; $\sigma t_r$, $\sigma t_\theta$, $\sigma t_z$, radial, tangential, and axial thermal stresses, respectively, kgf/$mm^2$.

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# STRUCTURAL AND MECHANICAL PROPERTIES AND EFFECTIVE PERMEABILITY OF FISSURED MATERIALS


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UDC 532.546



The article investigates the dependences of porosity and of the permeability tensor of an elastic fissured medium on the characteristics of its state of stress and the pressure of the permeating liquid.


Problems of permeation and convective transfer in deformed fissured and fractured-porous media are of interest in connection with the opening up of oil, gas condensate, and water deposits belonging to these types of collectors, and also for a number of problems of mining thermophysics. The structural method of describing motion in such media is based on the notions of continuity submitted in [1], and it has received fairly widespread application (see, e.g., [2-5]). Some inaccuracies characteristic of the continuous model in [2-5] were eliminated in [6]. In accordance with the model, the initial fractured-porous medium may be regarded as superposition of two coexisting porous continua modeling a system of interrelated cracks and a system of porous blocks.

The equations of motion in the mentioned continua contain as parameters fully characterizing the averaged properties of the medium: the effective porosity and permeability tensors referred to these continua, and also magnitudes describing the exchange of liquid between them. In regard to its meaning, the problem of determining the above parameters, which in the solution of various problems of permeation have to be regarded as known functions of the state of a medium filled with liquid, is completely analogous to the known rheological problem of the hydrodynamics of suspensions or the problem of determining the mechanical, thermophysical, and


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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp. 818-826, November, 1985. Original article submitted October 15, 1984.